

Identification of power distribution network topology via voltage correlation analysis

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Abstract—We consider the problem of reconstructing the topology of a portion of the power distribution network, given a dataset of voltage measurements. By using an approximate model for the grid voltage magnitudes, we show that these signals exhibit some specific correlation properties, that can be described via a sparse Markov random field. By specializing the tools available for the identification of graphical models, we propose an algorithm for the reconstruction of the grid topology. Via simulations, we show how the algorithm performs well also when an exact nonlinear model of the grid voltages is adopted, when realistic power demand profiles are considered, and when the voltage measurements are affected by measurement noise.

I. INTRODUCTION

Among the many R&D processes that are part of the development of a *smart* power grid [1], special attention is being recently paid to the deployment of information and communication technology (ICT) in the low and medium voltage power distribution network. The power distribution network, which corresponds to the portion of the power grid that delivers power from the high voltage transmission network to the users, has been excluded in the past from the deployment of communication infrastructures, automation, monitoring, and control. One notable exception is the capillary installation of smart metering devices, i.e. devices that can measure real time or aggregated power consumption at the level of the single user, can provide some basic grid monitoring and fault detection features, and could possibly convey real time energy prices to the users.

Different challenges are now emerging in the power distribution network, and are motivating a much deeper integration of information, communication, and control technology in this realm. For example, the large-scale penetration of distributed generation is posing both new opportunities and new issues. The possibility of generating power inside the power distribution grid (for example from solar panels, micro-wind installations, combined heat-and-power plants, microturbines) has the great advantage of exploiting cheaper and more sustainable energy sources, and to reduce power distribution losses because of the shortest path from the producer to the final consumer. However, distributed power generation, especially inside the highly resistive, radial, low voltage networks, can cause local overvoltage and power line congestions issues [2]. At the same time, a new generation of dispatchable loads are expected to appear very soon in the

power distribution network, e.g. hybrid electric vehicles, and today's power distribution grid will face major congestion issues if proper scheduling and coordination protocols will not be enforced to these consumers [3], [4].

Two main solutions are being currently investigated for these problems:

- **real time grid reconfiguration** - the topology of the power distribution grid can be partially reconfigured in order to avoid power line congestion, excessive voltage drops, and to reduce power losses; this option is well known in the power system operation practice [5], but its real-time automated implementation in the presence of the aforementioned new loads and new generation technologies is the subject of recent investigation [6];
- **real-time dispatchment, scheduling, and curtailment** - microgenerators, dispatchable loads, and storage devices can be commanded in order to minimize power distribution losses, avoid congestion, and ensure a safe and reliable operation of the power distribution grid; this optimal dispatchment can be enforced, or can emerge from a carefully designed energy market mechanism; in both cases, a distributed architecture for measurement, data processing, and actuation is needed [7], [8], [9]; this optimization process can also be applied to reactive power flows, as proposed for example in [10], [11], [12].

All these solutions require and assume that the topology of the power grid is known. While this is generally true in the high voltage grid, this is not always the case in power distribution networks. In many cases, the deployment of ICT in the power distribution grid will necessarily consist in the process of retro-fitting an existing infrastructure via the installation of new devices. Motivated by this scenario, *plug-and-play* approaches are often considered and may constitute in some cases the only viable solution. In *plug-and-play* approaches, the devices must identify the physical system in which they operate, discover their neighbor agents, and reconfigure the communication and control infrastructure in order to being able to perform the assigned tasks. Notice that, when the installed agents rely on some already available communication infrastructure (e.g. the Internet), the abstract communication layer provided by such infrastructure is in general of little help to identify the agent's neighborhood in the physical systems, and even to recognize the agents that belong to the same portion of the power distribution network, because this kind of information is lost in the protocols that are used in general purpose communication. In the special case in which *ad hoc* communication architectures

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are adopted (power line communication), the communication channel could instead be used to identify the grid topology, if some quite restrictive assumptions on the available bandwidth are satisfied [13].

We focus here on the specific problem of how the power distribution grid topology can be reconstructed from voltage measurements. We assume therefore that the agents (which could be, for example, the smart power meters) have no *a priori* information about how they are connected via the power lines, but they can measure the voltage magnitude at their point of connection. These measurements are collected over a period of time, processed, and finally an estimate of the network topology is inferred. Previous efforts in the estimation of the the power distribution grid topology have almost uniquely focused on the problem of estimating the effective impedance of the grid at the point of connection of the agents [14], [15]. Notice however that the result of such estimation procedures does not provide enough information to reconstruct what is the topology of the grid, which is the goal of the algorithm proposed in this paper.

The algorithm proposed in this paper is derived from the methodologies derived for the identification of Markov random fields (graphical models) [16], and is based on some conditional correlation properties that characterize voltage measurements in a radial grid. The use of these methods in the scenario of power networks has been explored recently with different goals: either the detection and localization of faults [17], or the detection of unmonitored switching of circuit breakers in the reconfiguration of the power distribution grid [18]. With respect to these works, it is also important to notice that, via an approximate solution for grid state adopted from [19], we do not rely on the hypothesis of purely inductive lines and we do not require synchronous voltage measurements via phasor measurement units (PMUs), which are seldom available in the power distribution grid.

The paper is organized in the following way. In Section II, the main notations are introduced. In Section III, a model for the power distribution grid is presented. In Section IV, the proposed identification algorithm is derived. Finally, in Section V, the algorithm is tested via simulations on a standard testbed, in order to check its performance and its robustness.

II. MATHEMATICAL PRELIMINARIES AND NOTATION

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \sigma, \tau)$ be a directed graph, where \mathcal{V} is the set of nodes, \mathcal{E} is the set of edges, and $\sigma, \tau : \mathcal{E} \rightarrow \mathcal{V}$ are two functions such that edge $e \in \mathcal{E}$ goes from the source node $\sigma(e)$ to the terminal node $\tau(e)$. With the notation $h \sim k$, we express the fact that an edge connects the two nodes h and k , regardless of the edge direction. We also define the set of neighbors of the node h as $\mathcal{N}(h) = \{k \in \mathcal{V} \mid h \sim k\}$.

In the next section we will introduce complex-valued functions defined on the nodes and on the edges. These functions will also be intended as vectors in \mathbb{C}^n (where $n = |\mathcal{V}|$) and $\mathbb{C}^{|\mathcal{E}|}$. Given a vector u , we denote by \bar{u} its element-wise complex conjugate, by $|u|$ its element-wise

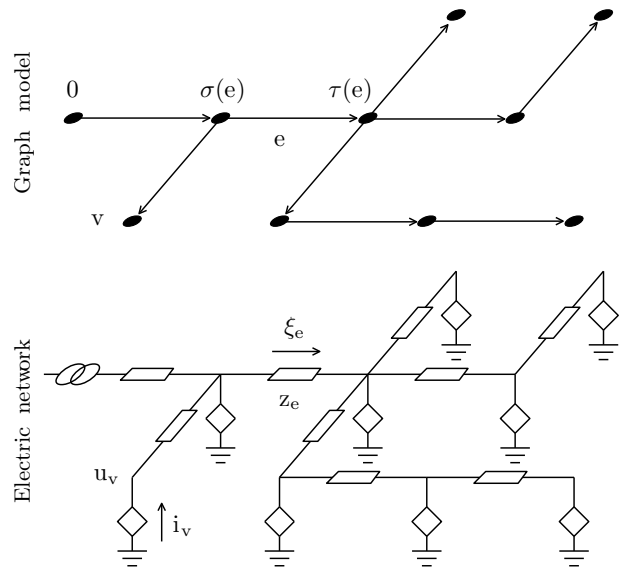


Fig. 1. Schematic representation of a portion of the power distribution network (lower panel) and of the corresponding graph model adopted in the paper (upper panel).

magnitude, and by u^T its transpose. We denote by $\Re(u)$ and by $\Im(u)$ the real and the imaginary part of u , respectively.

Let moreover $A \in \{0, \pm 1\}^{|\mathcal{E}| \times n}$ be the incidence matrix of the graph \mathcal{G} , defined via its elements

$$[A]_{ev} = \begin{cases} -1 & \text{if } v = \sigma(e) \\ 1 & \text{if } v = \tau(e) \\ 0 & \text{otherwise.} \end{cases}$$

If the graph \mathcal{G} is connected then $\mathbf{1}$ is the only vector in the null space $\ker A$, $\mathbf{1}$ being the column vector of all ones. We define by $\mathbf{1}_v$ the vector whose value is 1 in position v , and 0 everywhere else.

III. GRID MODEL

For the purpose of this paper, we model the power distribution network as a directed graph \mathcal{G} , in which edges represent the power lines, and nodes represent the devices that are connected to the grid (see Figure 1). These include the buses of the power distribution grid and also the point of connection to the transmission grid (the substation, also called point of common coupling, or PCC, and indexed as node 0).

We limit our study to the steady state behavior of the system, when all voltages and currents are sinusoidal signals at the same frequency. Each signal can therefore be represented via a complex number $y = |y|e^{j\angle y}$ whose absolute value $|y|$ corresponds to the signal root-mean-square value, and whose phase $\angle y$ corresponds to the phase of the signal with respect to an arbitrary global reference.

In this notation, the steady state of the grid is described by the following system variables (see Figure 1, lower panel):

- $u \in \mathbb{C}^n$, where u_v is the grid voltage at node v ;
- $i \in \mathbb{C}^n$, where i_v is the current injected by node v ;
- $\xi \in \mathbb{C}^{|\mathcal{E}|}$, where ξ_e is the current on the edge e .

For every edge e of the graph, we define by z_e the impedance of the corresponding power line. We assume the following.

Assumption 1: All power lines in the grid have the same inductance/resistance ratio, i.e.

$$z_e = e^{j\theta} |z_e|$$

for any e in \mathcal{E} and for a fixed θ .

This assumption is satisfied when the grid is relatively homogeneous, and is reasonable in most practical cases (see for example the IEEE standard testbeds [20]).

The following physical constraints are satisfied by the complex quantities u, i and ξ

$$A^T \xi + i = 0, \quad (1)$$

$$Au + e^{j\theta} Z \xi = 0, \quad (2)$$

where A is the incidence matrix of \mathcal{G} , and $Z = \text{diag}(|z_e|, e \in \mathcal{E})$ is the diagonal matrix of line impedances. Equation (1) corresponds to Kirchhoff's current law (KCL) at the nodes, while (2) describes the voltage drop on the edges of the graph. From (1) and (2) we can also obtain

$$i = e^{-j\theta} Lu \quad (3)$$

where $L \in \mathbb{R}^{n \times n}$ is the weighted Laplacian of the graph

$$L := A^T Z^{-1} A.$$

Each node v of the grid is also characterized by a law relating its injected current i_v with its voltage u_v . We model the PCC as an ideal sinusoidal voltage generator at the nominal voltage U_N (an infinite bus) with arbitrary, but fixed, angle ϕ

$$u_0 = U_N e^{j\phi}. \quad (4)$$

In the power system analysis terminology, node 0 is then a *slack bus* with fixed voltage magnitude and angle.

We model all the other nodes $v \in \mathcal{V} \setminus \{0\}$, except the PCC, as *constant power* or *P-Q buses*. Their voltage u_v and their current i_v must then satisfy

$$u_v \bar{i}_v = s_v, \quad \forall v \in \mathcal{V} \setminus \{0\}, \quad (5)$$

where s_v is the injected *complex power*.

The nonlinear equations (3), (4), and (5) implicitly define the power grid state. In the following, we present a convenient expression for the voltage magnitudes $|u_v|, v \in \mathcal{V}$ that will be useful for the analysis of the correlation properties of these signals. To do so, we need the following results, adopted from [19].

Lemma 2 (Lemma 1 in [19]): Let L be the weighted Laplacian of \mathcal{G} . There exists a unique symmetric, positive semidefinite matrix $X \in \mathbb{R}^{n \times n}$ such that

$$\begin{cases} XL = I - \mathbf{1}\mathbf{1}_0^T \\ X\mathbf{1}_0 = 0. \end{cases} \quad (6)$$

Lemma 3 (Proposition 1 in [19]): Consider the physical model described by the set of nonlinear equations (3), (4), and (5). Node voltages then satisfy

$$u = e^{j\phi} \left(U_N \mathbf{1} + \frac{e^{j\theta}}{U_N} X \bar{s} + \frac{c(U_N)}{U_N^2} \right), \quad (7)$$

where the complex valued function $c(U_N)$ is bounded for $U_N \rightarrow \infty$, and where s is the complex vector containing the nominal powers $s_v, v \in \mathcal{V} \setminus \{0\}$, together with $s_0 = 0$.

The term $\frac{c(U_N)}{U_N^2}$ is negligible if the grid is operated at large nominal voltage U_N and relatively small currents supplied to the loads. This assumption is verified in practice, and corresponds to correct design and operation of power distribution networks, where indeed the nominal voltage is chosen sufficiently large (subject to other functional constraints) in order to deliver electric power to the loads with relatively small power losses on the power lines. In [19], a brief discussion about how this approximation extends the DC power flow model [21, Chapter 3] to the lossy case, has been provided.

Based on this approximation, we have the following expression for the voltage magnitudes that can be measured at every bus. The proof is omitted due to space constraints, and can be found in the online extended version of the paper [22].

Lemma 4: Consider the physical model described by the set of nonlinear equations (3), (4), and (5). The vector $|u|$ of voltage magnitudes $|u_v|, v \in \mathcal{V}$, then satisfies

$$|u| = U_N \mathbf{1} + \frac{1}{U_N} \Re(e^{j\theta} X \bar{s}) + \frac{d(U_N)}{U_N^2},$$

where the real valued function $d(U_N)$ is bounded for $U_N \rightarrow \infty$, and where s is the complex vector containing the nominal powers $s_v, v \in \mathcal{V} \setminus \{0\}$, together with $s_0 = 0$.

IV. IDENTIFICATION ALGORITHM

In order to derive an algorithm that is capable of reconstructing the power distribution grid topology from the voltage measurements collected on the grid, we first provide an analysis of the correlation properties of such measurements.

In order to do so, we introduce the following assumption.

Assumption 5: At every time t , the active power demands $p_v, v \in \mathcal{V}$, of the individual nodes are uncorrelated random variables with finite variance $\sigma_{p_v}^2$. Similarly, the reactive power demands $q_v, v \in \mathcal{V}$, of the individual nodes are uncorrelated random variables with finite variance $\sigma_{q_v}^2$.

This assumption is a simplification of the behavior of individual demands in the power distribution grid. As pointed out in [23] (and references therein) and in [24], these signals exhibit a mutual correlation because of a number of coupling factors (outside temperature, destination of use of the buildings, time of the day, etc.). However, on a short time scale, and when the nodes of the grid correspond to the aggregation of a small number of intermittent loads, the correlation of the instantaneous power demands is limited. The robustness of the solution proposed in this paper with respect to this approximation has been addressed in the simulations (where

the simulated power demands exhibit some correlation) and is subject of current research.

The covariance matrix of the signal $|u|$ can be computed according to the expression proposed in Lemma 4, neglecting the infinitesimal terms and obtaining

$$\Lambda := \mathbb{E} \left[(|u| - \mathbb{E}|u|) (|u| - \mathbb{E}|u|)^T \right] = \frac{1}{U_N^2} X \Sigma X,$$

where we used the fact that $\Re(e^{j\theta} X \bar{s}) = X(\cos \theta p + \sin \theta q)$ and we defined the diagonal covariance matrix

$$\Sigma = \cos \theta \Sigma^P + \sin \theta \Sigma^Q,$$

where $\Sigma^P = \text{diag}(\sigma_{p_v}^2, v \in \mathcal{V})$ and $\Sigma^Q = \text{diag}(\sigma_{q_v}^2, v \in \mathcal{V})$.

Notice that, because $s_0 = 0$, the matrix Σ is not invertible and satisfies $\Sigma \mathbf{1}_0 = 0$. Moreover, because X also satisfies $X \mathbf{1}_0 = 0$ and is symmetric, we have that also Λ is singular and satisfies $\Lambda \mathbf{1}_0 = 0$.

We introduce the following pseudoinverse of Λ .

Lemma 6: Let Λ be a positive semidefinite matrix that satisfies $\Lambda \mathbf{1}_0 = 0$. There exists a unique symmetric, positive semidefinite matrix $K \in \mathbb{R}^{n \times n}$ such that

$$\begin{cases} K \Lambda = I - \mathbf{1}_0 \mathbf{1}^T \\ K \mathbf{1} = 0. \end{cases} \quad (8)$$

The proof is omitted due to space constraints, and can be found in the online extended version of the paper [22].

The matrix K is known as *concentration matrix*, and has an interesting and well known interpretation in terms of conditional correlation. In particular, when K is invertible, $K_{hk} = 0$ if and only if $|u_h|$ and $|u_k|$ are conditionally uncorrelated given all other voltages $|u_\ell|$, $\ell \neq h, k$. Notice that the fact that $|u_0|$ is the zero random variable makes K singular, and makes the problem of determining its conditional correlation ill posed. The particular choice of the K pseudoinverse comes into help and yields some interesting properties, as we will show next.

The matrix K can also be expressed as an algebraic function of the grid Laplacian matrix and of the covariance matrix of the power demands, in the form

$$K = U_N^2 L \Sigma^\dagger L \quad (9)$$

where Σ^\dagger is the diagonal pseudoinverse of Σ defined as

$$\Sigma_{vv}^\dagger = \begin{cases} 0 & \text{if } \Sigma_{vv} = 0 \\ (\Sigma_{vv})^{-1} & \text{if } \Sigma_{vv} \neq 0. \end{cases}$$

Notice that Σ^\dagger has the same kernel of Σ , i.e. $\Sigma \mathbf{1}_0 = 0$.

Conditions of Lemma 6 can be verified by inspection, as

$$K \mathbf{1} = U_N^2 L \Sigma^\dagger L \mathbf{1} = 0$$

and

$$\begin{aligned} K \Lambda &= L \Sigma^\dagger L X \Sigma X = L \Sigma^\dagger (I - \mathbf{1}_0 \mathbf{1}^T) \Sigma X = \\ &= L \Sigma^\dagger \Sigma X = I - \mathbf{1}_0 \mathbf{1}^T, \end{aligned}$$

where in the last step we used the fact that $\Sigma \Sigma^\dagger$ is a projector operator on the subspace orthogonal to $\mathbf{1}_0$.

It is clear, by looking at (9), that the matrix K bears some useful information about the sparsity pattern of L . In particular, the following result shows how K has the sparsity pattern of L^2 , and because of the tree structure of the graph \mathcal{G} , it also exhibits an useful sign pattern. The proof is omitted due to space constraints, and can be found in the online extended version of the paper [22].

Proposition 7: Let K be defined as in (9). Let $h, k \in \mathcal{V}$ be two nodes in \mathcal{G} . We then have

$$K_{hk} \begin{cases} > 0 & \text{if } h = k \\ < 0 & \text{if } h \sim k \\ > 0 & \text{if } \exists \ell \in \mathcal{V} \text{ such that } h \sim \ell \text{ and } \ell \sim k \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 7 therefore shows that the sparsity pattern of K is the same sparsity pattern of the L^2 . In the terminology of Markov random fields, this means that the corresponding graphical model is an undirected graph in which nodes are connected by an edge (and therefore they are conditionally correlated) if they are 1-hop or 2-hop neighbors in the graph \mathcal{G} describing the power distribution grid lines.

Proposition 7 also shows that the strictly negative elements of K have the sparsity pattern of L and can therefore be directly used to reconstruct the topology of the graph \mathcal{G} .

Based on these results, we propose the following algorithm for the identification of the grid topology, given a sequence of measurements $|u|^{(t)}$, $t = 1, \dots, N$ collected on the grid.

Algorithm

- 1) Compute the sample covariance matrix $\hat{\Lambda} = \text{cov}(|u|^{(t)}, t = 1, \dots, N)$.
- 2) Compute the matrix \hat{K} that satisfies

$$\begin{aligned} \hat{K} \hat{\Lambda} &= I - \mathbf{1}_0 \mathbf{1}^T \\ \hat{K} \mathbf{1} &= 0. \end{aligned}$$

- 3) Consider the complete graph $\mathcal{G}_{\hat{K}}$ defined on the nodes of the grid \mathcal{V} , with edge weights described by the elements of \hat{K} . Compute the minimum spanning tree on $\mathcal{G}_{\hat{K}}$, i.e. the subgraph of $\mathcal{G}_{\hat{K}}$ that is a tree, connects all the nodes, and whose total edge cost is less or equal to any other spanning tree.

Notice that the second step of the algorithm can be computed numerically via the matrix inversion (??), and that the minimum spanning tree can be computed in polynomial time by greedy algorithms like the Prim's algorithm [25].

The proposed algorithm resembles, in some sense, the well known Chow-Liu algorithm [26] for graphical model identification, in which however the choice of the best spanning tree is motivated by the search for the closest approximation of the actual distribution in an information-theoretic sense. In our scenario, on the other hand, we know in advance that there exists a tree which is the *root* (in the graph-theory sense) of the graph that describes the actual distribution (i.e. the actual graph connects nodes that can be reached in 1 or 2 hops in such tree), and we make explicit

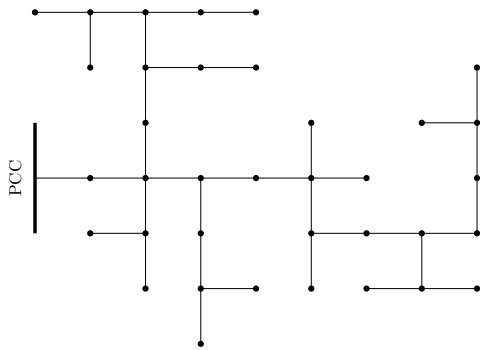


Fig. 2. Schematic representation of the IEEE37 test feeder.

use of this additional information. Because of this *a priori* knowledge, we also don't need the tools that have been developed for model selection [27] (i.e., in some sense, to tune the sparsity level of the estimated graph).

V. SIMULATIONS

In order to validate the proposed algorithm, we considered the standard medium voltage testbed IEEE 37 [20]. This testbed is an actual portion of 4.8kV radial power distribution network located in California. The load buses are a blend of constant-power, constant-current, and constant-impedance loads, with a total power demand of almost 2 MW of active power and 1 MVAR of reactive power (see [20] for the testbed data). The length of the power lines range from a minimum of 25 meters to a maximum of almost 600 meters. The impedance of the power lines differs from edge to edge (for example, resistance ranges from 0.182 Ω/km to 1.305 Ω/km). However, the inductance/resistance ratio exhibits a smaller variation, ranging from $\angle z_e = 0.47$ to $\angle z_e = 0.59$. This justifies Assumption 1, in which we claimed that $\angle z_e$ can be considered constant across the network.

A time-varying profile for the loads has been generated, in order to simulate the effect of slowly varying loads (e.g. aggregate residential demand), fast changing demands (e.g. some industrial loads), intermittent large loads (e.g. heating), and correlated generation (e.g. solar panels).

Voltage measurements have been generated by solving the system of nonlinear equations of the grid, via a numerical solver [28]. The approximate model presented before has not been used in the simulations.

We considered measurement errors complying with three different IEC accuracy classes: 0.1, 0.2, and 0.5 (where the number refers to the maximum % voltage measurement error). These are typical levels of accuracy that can be found in off-the-shelf devices for power system applications.

Figure 3 illustrates the value of the elements of the concentration matrix \hat{K} . Blue squares indicate negative elements, while red square indicate positive elements. Because the matrix is obtained from noisy data, the matrix it is not sparse. The dots inside the squares indicate the non-zero elements of the Laplacian, and according to the analysis proposed in this paper, should correspond to negative elements of \hat{K} . The figure shows the importance of the minimum spanning

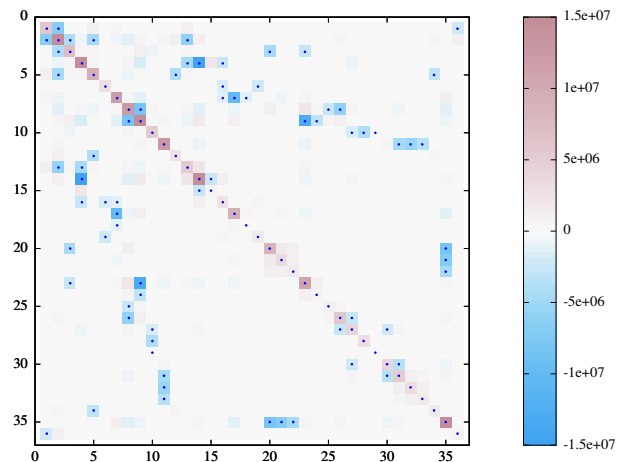


Fig. 3. A colormap representation of the estimated concentration matrix \hat{K} , together with the sparsity pattern of L , indicated by small dots.

tree algorithm for the identification of the correct edges to include in the estimated topology. In this specific realization, the algorithm has been able to identify the correct topology. It can be seen how a simpler threshold analysis (i.e. finding a threshold τ and imposing that all pair of nodes h, k such that $\hat{K}_{hk} < \tau$ are connected) would not have worked in this case: for example, we have that $\hat{K}_{4,9} < \hat{K}_{10,29}$, even if the first does not correspond to an edge of the graph, while the second does. However, the element $\hat{K}_{10,29}$ is the smallest among all elements $\hat{K}_{h,29}$, $h \in \mathcal{V}$, and is therefore chosen by the minimum spanning tree algorithm as the most likely edge that connects node 29 to the rest of the graph.

Figure 4 illustrate the performance of the algorithm for different numbers of collected samples and for different measurement accuracy classes. In particular, we have reported the probability of error in the reconstruction of the correct graph topology. The estimated error probability have been reported in the table, and in the plot as thick lines. The confidence intervals in the plot have been computed according to the Wilson score interval [29].

It can be seen that, even in the presence of a number of non-idealities, the algorithm is capable of reconstructing the grid topology, given a large enough number of measurements and a minimal accuracy in the measuring devices.

VI. CONCLUSIONS

In this work, we tackled the novel problem of topology estimation in power distribution networks, which is a preliminary and necessary step for enabling the incorporation of ICT solutions in this realm. We proposed an algorithm that, given voltage amplitude measurements performed at the nodes of the grid, can estimate the grid topology from a correlation analysis of such signals. We proved via simulations that the algorithm is indeed effective in reconstructing the correct topology, provided that the measurement noise is limited that the number of samples is sufficiently large.

The next steps will necessarily be the derivation of a formal analysis of the properties of this algorithm, based on the many results that are available for loop-less graphical

meas. err.	Number of measurements				
	360	720	1440	2880	5760
0.1	65.8%	22.9%	2%	0.025%	0%
0.2	91.1%	55.8%	18.95%	4.05%	0.35%
0.5	100%	100%	99%	90%	72%

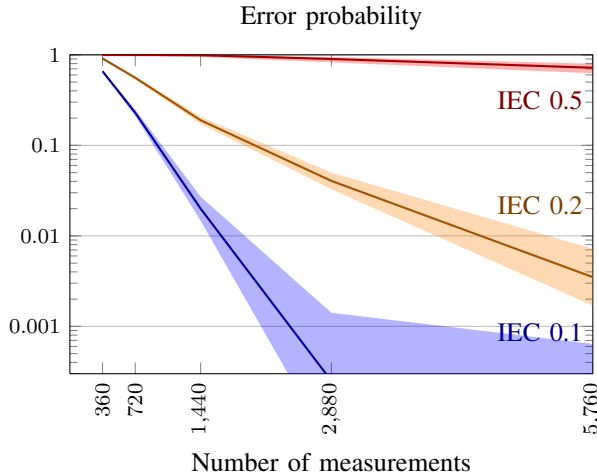


Fig. 4. Algorithm performance evaluated via a series of Monte Carlo simulations, for different measurement noises and different data sizes. The estimates of the error probability have been reported in the table, and in the plot as thick lines, together with their confidence intervals.

models. Moreover, the authors are interested in extending the same approach to the practical case in which not all the nodes are monitored, but only a limited number of intelligent agent are deployed. This study will likely require the adoption of some methodologies derived for hidden variables in Markov random fields. Finally, it will be investigated if the possibility of measuring also the power injections at the nodes (i.e. the input noise in the Markov random field, which is unknown in typical formulations) can be used to estimate also other relevant parameters of the grid lines, and in particular the impedance of the individual edges.

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