Virtual Inertia Emulation and Placement in Power Grids

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At the beginning of power systems was...

At the beginning was the **synchronous machine**:

\[ M \frac{d}{dt} \omega(t) = P_{\text{generation}}(t) - P_{\text{demand}}(t) \]

change of kinetic energy = instantaneous power balance

**Fact:** the AC grid & all of power system operation has been designed around synchronous machines.

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Operation centered around bulk synchronous generation

Distributed/non-rotational/renewable generation on the rise

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Source: W. Sattinger, Swissgrid

Source: Renewables 2014 Global Status Report
A few (of many) game changers . . .

synchronous generator new workhorse scaling

location & distributed implementation

Almost all operational problems can principally be resolved . . . but one (?)

Low-inertia stability: # 1 problem of distributed generation

# frequency violations in Nordic grid (source: ENTSO-E)
same in Switzerland (source: Swissgrid)

inertia is shrinking, time-varying, & localized, . . . & increasing disturbances

Solutions in sight: none really . . . other than emulating virtual inertia through fly-wheels, batteries, super caps, HVDC, demand-response, . . .

Fundamental challenge: operation of low-inertia systems

We slowly lose our giant electromechanical low-pass filter:

$$M \frac{d}{dt} \omega(t) = P_{\text{generation}}(t) - P_{\text{demand}}(t)$$

cchange of kinetic energy = instantaneous power balance

Virtual inertia emulation

devices commercially available, required by grid-codes or incentivized through markets

⇒ plug-&-play (decentralized & passive), grid-friendly, user-friendly, . . .
⇒ today: where to do it? how to do it properly?
Inertia emulation & virtual synchronous machines

1. naive D-control on $\omega(t)$: $M \frac{d\omega(t)}{dt} = P_{\text{generation}}(t) - P_{\text{demand}}(t)$

2. more sophisticated emulation of virtual synchronous machine

- everything in between ... and much more ...
  - by measuring AC current/voltage/power/frequency
  - software model of virtual machine provides converter setpoints
  - actuation via modulation (switching) or DC injection (batteries etc.)
Challenges in real-world converter implementations

1. delays in measurement acquisition, signal processing, & actuation
2. accuracy in AC measurements (averaged over $\approx 5$ cycles)
3. constraints on currents, voltages, power, etc.
4. guarantees on stability and robustness

**today:** use DC measurement, exploit analog storage, & passive control

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**standard** power electronics control would continue by
1. constructing voltage/current/power references (e.g., droop, synchronous machine emulation, etc.)
2. tracking these references at the converter terminals typically by means of cascaded PI controllers

let’s do **something different** (smarter?) today . . .
Model matching (≠ emulation) as inner control loop

DC cap & AC filter equations:
\[ C_{dc} v_{dc} = -G_{dc} v_{dc} + i_{dc} - \frac{1}{2} m^T i_{\alpha\beta} \]
\[ C v_{\alpha\beta} = -i_{\text{load}} + i_{\alpha\beta} \]
\[ L i_{\alpha\beta} = -R i_{\alpha\beta} + \frac{1}{2} m v_{dc} - v_{\alpha\beta} \]

**Matching control**: \( \dot{\theta} = \eta \cdot v_{dc}, \quad m = \mu \cdot \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix} \) with \( \eta, \mu > 0 \)

**Pros**: is balanced, uses natural storage, & based on DC measurement

**Virtual machine** with \( M = \frac{C_{dc}}{\eta^2}, \quad D = \frac{G_{dc}}{\eta^2}, \quad \tau_m = \frac{i_{\text{dc}}}{\eta}, \quad i_f = \frac{\mu}{\eta L_m} \)

**Base for outer controls** via \( i_{dc} & \mu \), e.g., virtual torque, PSS, & inertia

Some properties & different viewpoints

- **Quadratic curves** for stationary \( P \) vs. \( (|V|, \omega) \)
  \[ P \leq P_{\text{max}} = \frac{i_{dc}^2}{4 G_{dc}} \]
- Reactive power not directly affected
- \( (P, \omega) \)-droop \( \approx \frac{1}{\eta} \)
- \( (P, |V|) \)-droop \( \approx \frac{1}{\mu} \)
- **Reformulation as virtual & adaptive oscillator**
- **Remains passive**: \( (i_{dc}, i_{\text{load}}) \rightarrow (v_{dc}, v_{\alpha\beta}) \)

**Optimal placement of virtual inertia**
Linearized & Kron-reduced swing equation model

\[ m_i \ddot{\theta}_i + d_i \dot{\theta}_i = p_{in,i} - p_{e,i} \]

\( p_{e,i} \approx \sum_{j \in N} b_{ij} (\theta_i - \theta_j) \)

Generator swing equations

State space representation:

\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\omega}
\end{bmatrix} =
\begin{bmatrix}
0 & I \\
-M^{-1}L - M^{-1}D & A
\end{bmatrix}
\begin{bmatrix}
\theta \\
\omega
\end{bmatrix} +
\begin{bmatrix}
0 \\
M^{-1}
\end{bmatrix}
\begin{bmatrix}
T^{1/2} \eta
\end{bmatrix}
\]

where \( M = \text{diag}(m_i) \), \( D = \text{diag}(d_i) \), \( T = \text{diag}(t_i) \), & \( L = L^T \) (Laplacian)

Coherency performance metric & \( \mathcal{H}_2 \) norm

Energy expended by the system to return to synchronous operation:

\[
\int_0^\infty \sum_{(i,j) \in \mathcal{E}} e^{a_{ij}(\theta_i(t) - \theta_j(t))^2} + \sum_{i=1}^n s_i \omega_i^2(t) \, dt
\]

\( \mathcal{H}_2 \) norm interpretation:

- Associated performance output:
  \[
  y = \begin{bmatrix}
  Q_1^{1/2} & 0 \\
  0 & Q_2^{1/2}
  \end{bmatrix}
  \begin{bmatrix}
  \theta \\
  \omega
  \end{bmatrix}
  \]

- Impulses (faults) \( \rightarrow \) output energy \( \int_0^\infty y(t)^T y(t) \, dt \)

- White noise (renewables) \( \rightarrow \) output variance \( \lim_{t \to \infty} \mathbb{E} (y(t)^T y(t)) \)

Algebraic characterization of the \( \mathcal{H}_2 \) norm

Lemma: via observability Gramian

\[
\|G\|_2^2 = \text{Trace}(B^T P B)
\]

where \( P \) is the observability Gramian \( P = \int_0^\infty e^{A^T t} C^T C e^A t \, dt \)

- \( P \) solves a Lyapunov equation: \( PA + A^T P + Q = 0 \)

- \( A \) has a zero eigenvalue \( \rightarrow \) restricts choice of \( Q \)

\[
\begin{bmatrix}
Q_1^{1/2} & 0 \\
0 & Q_2^{1/2}
\end{bmatrix}
\begin{bmatrix}
\theta \\
\omega
\end{bmatrix} = 0
\]

- \( P \) is unique for \( P [\mathbb{1} 0] = [0 0] \)
Problem formulation

\[ \minimize_{P, m_i} \| G \|_2^2 = \text{Trace}(B^T PB) \rightarrow \text{performance metric} \]

subject to \[ \sum_{i=1}^n m_i \leq m_{bdg} \rightarrow \text{budget constraint} \]

\[ m_i \leq m_i \leq m_i, \quad i \in \{1, \ldots, n\} \rightarrow \text{capacity constraint} \]

\[ PA + A^T P + Q = 0 \rightarrow \text{observability Gramian} \]

\[ P [1 \ 0] = [0 \ 0] \rightarrow \text{uniqueness} \]

Insights

- \( m \) appears as \( m^{-1} \) in system matrices \( A, B \)
- product of \( B(m) \) & \( P \) in the objective
- product of \( A(m) \) & \( P \) in the constraint

\[ \Rightarrow \text{large-scale & non-convex} \]

Closed-form results for cost of primary control

\( P/\dot{\theta} \) primary droop control

\[ (\omega_i - \omega^*) \propto (P_i^* - P_i(\theta)) \]

\[ D_i \dot{\theta}_i = P_i^* - P_i(\theta) \]

Primary control effort \( \rightarrow \) accounted for by integral quadratic cost

\[ \int_0^\infty \dot{\theta}(t)^T D \dot{\theta}(t) \, dt \]

which is the \( \mathcal{H}_2 \) performance for the penalties \( Q_1^{1/2} = 0 \) and \( Q_2^{1/2} = D \)

Building the intuition: results for two-area networks

Fundamental learnings

- explicit closed-form solution is rational function
- sufficiently uniform \( (t/d)_i \) \( \rightarrow \) strongly convex & fairly flat cost
- non trivial in the presence of capacity constraints

Primary Control \( \ldots \) cont’d

**Theorem:** the primary control effort optimization reads equivalently as

\[ \minimize_{m_i} \sum_{i=1}^n t_i/m_i \]

subject to \[ \sum_{i=1}^n m_i \leq m_{bdg} \]

\[ m_i \leq m_i \leq m_i, \quad i \in \{1, \ldots, n\} \]

**Key take-aways:**

- optimal solution independent of network topology
- allocation \( \propto \sqrt{t_i} \) or \( m_i = \min\{m_{bdg}, m_i\} \)

Location & strength of disturbance are crucial solution ingredients
Taylor & power series expansions

**Key idea:** expand the performance metric as a power series in \( m \)

\[
\|G\|_2^2 = \text{Trace}(B(m)^T P(m) B(m))
\]

**Motivation:** scalar series expansion at \( m_i \) in direction \( \mu_i \):

\[
\frac{1}{(m_i + \delta \mu_i)} = \frac{1}{m_i} - \frac{\delta \mu_i}{m_i^2} + O(\delta^2)
\]

Expand system matrices as **Taylor series** in direction \( \mu \):

\[
A(m + \delta \mu) = A(0)_{(m,\mu)} + A(1)_{(m,\mu)} \delta + O(\delta^2)
\]

\[
B(m + \delta \mu) = B(0)_{(m,\mu)} + B(1)_{(m,\mu)} \delta + O(\delta^2)
\]

Expand the observability Gramian as a **power series** in direction \( \mu \):

\[
P(m + \delta \mu) = P(0)_{(m,\mu)} + P(1)_{(m,\mu)} \delta + O(\delta^2)
\]

Explicit gradient computation

Expansion of system matrices & Gramian \( \Rightarrow \) **match coefficients** ... 

**Formula for gradient at \( m \) in direction \( \mu \)**

- nominal Lyapunov equation for \( O(\delta^0) \):
  \[
P(0) = \text{Lyap}(A(0),Q)
  \]

- perturbed Lyapunov equation for \( O(\delta^1) \) terms:
  \[
P(1) = \text{Lyap}(A(0),P(0)A(1) + A(1)^T P(0))
  \]

- expand objective in direction \( \mu \):
  \[
  \|G\|_2^2 = \text{Trace}(B(m)^T P(m) B(m)) = \text{Trace}(... + \delta(...)) + O(\delta^2)
  \]

- gradient: \( \text{Trace}(2 \ast B(1)^T P(0) B(0) + B(0)^T P(1) B(0)) \)

\( \Rightarrow \) use favorite method for reduced optimization problem

results
Modified Kundur case study: 3 regions & 12 buses
transformer reactance 0.15 p.u., line impedance (0.0001+0.001i) p.u./km

Heuristics outperformed by $H_2$ - optimal allocation

Scenario: disturbance at #4
- locally optimal solution outperforms heuristic max/uniform allocation
- optimal allocation ≈ matches disturbance
- inertia emulation at all undisturbed nodes is actually detrimental
⇒ location of disturbance & inertia emulation matters

Eye candy: time-domain plots of post fault behavior

Take-home messages:
- best oscillation performance
- smallest peak frequency at #4
- undisturbed sites are irrelevant
- minimal control effort $m_i \cdot \dot{\theta}_i$

conclusions
Conclusions on virtual inertia emulation

Where to do it?
1. $H_2$-optimal (non-convex) allocation
2. closed-form results for cost of primary control
3. numerical approach via gradient computation

How to do it?
1. down-sides of naive inertia emulation
2. novel machine matching control

What else to do? Inertia emulation is . . .

- decentralized, plug-and-play (passive), grid-friendly, user-friendly, . . .
- suboptimal, wasteful in control effort, & need for new actuators

Recall: operation centered around (virtual) sync generators

A control perspective of power system operation

Conventional strategy: emulate generator physics & control

\[
M \dot{\omega}(t) = P_{\text{mech}} - D \omega(t) - \int_0^t \omega(\tau) \, d\tau - P_{\text{elec}}
\]

(provisional inertia)  (tertiary control)  (primary control)  (secondary control)

Essentially all PID + setpoint control (simple, robust, & scalable)

\[
M \dot{\omega}(t) = P - D \omega(t) - \int_0^t \omega(\tau) \, d\tau - P_{\text{elec}}
\]

D  (set-point)  P  I

Control engineers should be able to do better . . .

This “what else?” has been broadly recognized
by TSOs, device manufacturers, academia, etc.

Massive InteGRATion of power Electronic devices

“The question that has to be examined is: how much power
electronics can the grid cope
with?” (European Commission)

current controls  what else?

all options are on the table and keep us busy . . .
Spectral perspective on different inertia allocations

Cone, Original, Optimal, and Uniform allocations

\[ \begin{align*}
\text{Cone}, & \quad \text{Original}, \quad \text{Optimal}, \quad \text{and Uniform allocations} \\
\end{align*} \]

The planning problem

sparse allocation of limited resources

\[ \begin{align*}
\ell_1\text{-regularized inertia allocation (promoting a sparse solution):} \\
\text{minimize} & \quad J_{\gamma}(m, P) = \|G\|_2^2 + \gamma \|m - \bar{m}\|_1 \\
\text{subject to} & \quad \sum_{i=1}^{n} m_i \leq m_{\text{bdg}} \\
& \quad m_i \leq m_i \leq \bar{m}_i \quad i \in \{1, \ldots, n\} \\
& \quad PA + A^TP + Q = 0 \\
& \quad P[1 0] = [0 0] \\
\end{align*} \]

where $\gamma \geq 0$ trades off sparsity penalty and the original objective

Highlights:

- $m = \bar{m} \rightarrow$ best damping asymptote & best damping ratio
- Spectrum holds only partial information !!
Relative performance loss (%) as a function of $\gamma$

0% $\rightarrow$ optimal allocation, 100% $\rightarrow$ no additional allocation

![Cardinality vs Relative Performance Loss](image)

- uniform disturbance $\Rightarrow \exists \gamma$ 1.3% loss $\equiv (9 \rightarrow 7)$
- localized disturbance $\Rightarrow (2 \rightarrow 1)$ without affecting performance

**Uniform disturbance to damping ratio**

Power sharing $\rightarrow d \propto P^*$, assuming $t \propto$ source rating $P^*$

**Theorem:** for $t_i/d_i = t_j/d_j$ the allocation problem reads equivalently as

$$\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{n} \frac{s_i}{m_i} \\
\text{subject to} & \quad \sum_{i=1}^{n} m_i \leq m_{bgj}, \quad m_i \leq m_i, \quad i \in \{1, \ldots, n\}
\end{align*}$$

**Key takeaways:**
- optimal solution independent of network topology
- allocation $\propto \sqrt{s_i}$ or $m_i = \min\{m_{bgj}, m_i\}$

What if freq. penalty $\propto$ inertia? $\rightarrow$ norm independent of inertia

**Taylor & power series expansions**

**Key idea:** expand the performance metric as a power series in $m$

$$\|G\|^2 = \text{Trace}(B(m)^T P(m) B(m))$$

**Motivation:** scalar series expansion at $m_i$ in direction $\mu_i$:

$$\frac{1}{(m_i + \delta\mu)} = \frac{1}{m_i} - \frac{\delta\mu}{m_i^2} + O(\delta^2)$$

**Expand** system matrices in direction $\mu$, where $\Phi = \text{diag}(\mu)$:

$$\begin{align*}
A_{(m,\mu)}^{(0)} &= \begin{bmatrix} 0 & I \\ -M^{-1}L & -M^{-1}D \end{bmatrix}, & A_{(m,\mu)}^{(1)} &= \begin{bmatrix} 0 & 0 \\ \Phi M^{-2}L & \Phi M^{-2}D \end{bmatrix} \\
B_{(m,\mu)}^{(0)} &= \begin{bmatrix} 0 \\ M^{-1}T^{1/2} \end{bmatrix}, & B_{(m,\mu)}^{(1)} &= \begin{bmatrix} 0 \\ -\Phi M^{-2}T^{1/2} \end{bmatrix}
\end{align*}$$

**Expand** the observability Gramian as a power series in direction $\mu$.

$$P(m) = P(m + \delta\mu) = P_{(m,\mu)}^{(0)} + P_{(m,\mu)}^{(1)} \delta + O(\delta^2)$$

**Formula for gradient in direction $\mu$**

- nominal Lyapunov equation for $O(\delta^2)$: $P^{(0)} = \text{Lyap}(A^{(0)}, Q)$
- perturbed Lyapunov equation for $O(\delta^2)$ terms:
  $$P^{(1)} = \text{Lyap}(A^{(0)}, P^{(0)} A^{(1)} + A^{(1)}^T P^{(0)})$$
- expand objective in direction $\mu$:
  $$\|G\|^2 = \text{Trace}(B(m)^T P(m) B(m)) = \text{Trace}(\ldots + \delta(\ldots)) + O(\delta^2)$$
- gradient: $\text{Trace}(2 \ast B^{(1)^T} P^{(0)} B^{(0)} + B^{(0)^T} P^{(1)} B^{(0)})$
Gradient computation

**Algorithm:** Gradient computation & perturbation analysis

**Input** → current values of the decision variables $m_i$

**Output** → numerically evaluated gradient $\nabla f$ of the cost function

- Evaluate the system matrices $A^{(0)}, B^{(0)}$ based on current inertia
- Solve for $P^{(0)} = \text{Lyap}(A^{(0)}, Q)$ using a Lyapunov routine
- For each node- obtain the perturbed system matrices $A^{(1)}, B^{(1)}$
- Compute $P^{(1)} = \text{Lyap}(A^{(0)}, P^{(0)}A^{(1)} + A^{(1)}^TP^{(0)})$
- Gradient $\Rightarrow \text{Trace}(2B^{(1)^TP^{(0)}B^{(0)} + B^{(0)^TP^{(1)}B^{(0)}}})$

**Heuristics outperformed also for uniform disturbance**

**Scenario:** uniform disturbance

**Heuristics** for placement:
- **max** allocation in case of capacity constraints
- **uniform** allocation in case of budget constraint

**Results & insights:**
- locally optimal solution **outperforms** heuristics
- optimal solution $\neq \text{max}$ inertia at each bus