Control of Low-Inertia Power Grids: A Model Reduction Approach

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Abstract— **A major transition in the operation of electric power grids is the replacement of conventional power generation using synchronous machines by distributed generation based on renewable sources interfaced by power electronics. In contrast to synchronous machines, which stabilize the power system through a combination of their inherent physical properties and their controls, power converters do not inherently stabilize the power system. Moreover, the models used for grid-level stability analysis also crucially depend on the properties of synchronous machines. As a first step toward addressing these challenges, we propose a novel reduced-order model for analysis and control design of low-inertia power systems. Starting from a detailed nonlinear first-principle model of a low-inertia power system, including detailed power converter models and their interactions with the power grid, we use arguments from singular perturbation theory to obtain a tractable model for control design. We use insights gained from the reduced model to bridge the gap between grid-level objectives and device-level control by introducing an internal model and matching controller that exploits structural similarities between power converters and synchronous generators. Moreover, we propose a nonlinear droop control that stabilizes the power system.**

I. Introduction

Tomorrow's electric power generation and transmission is envisioned to be clean, sustainable, and largely based on renewable sources interfaced with power electronics. In contrast, today's power system heavily relies on conventional power plants with synchronous generators, whose inherent physical properties are the robust foundation of today's power grid. In particular, their rotational inertia and their controls ensure stability of the power grid [1]. As renewable generation replaces conventional generation, this foundation and safeguard of today's power system is replaced by fluctuating renewable sources. This results in larger and more frequent frequency deviations and jeopardizes the stability of the power grid [2]. At the same time, the analysis of such phenomena is a challenging problem because the power system physics are highly nonlinear, large-scale, and contain dynamics on multiple time scales from mechanical and electrical domains. As a result, the analysis and control of conventional power system is typically based on reduced-order models of various degrees of fidelity [1].

A widely accepted reduced-order model of conventional power systems is a structure-preserving multi-machine model, where each generator model is reduced to the swing equation describing the interaction between the generator rotor and the grid, which is itself modeled at quasi-steady state via the nonlinear algebraic power balance equations [1]. While this prototypical model has proved itself useful its validity for conventional power systems has always been a subject of debate; see [3], [4] for recent discussions. Because this model crucially relies on inherent physical properties of synchronous generators its validity for low-inertia power systems is questionable. In particular, if power electronics are modeled as a constant power sources and all generators are removed, then only purely algebraic equations remain.

One approach to mitigate the loss of rotational inertia and, at the same time, salvage the current tools for *system-level* analysis is to use power electronic devices to emulate the inherent behavior of synchronous generators to various degrees of fidelity [5], [6]. However, this *device-level* emulation is subject to non-negligible measurement delays due to signal processing, e.g., phase locked loops, which are known to have deteriorating effects on the *system level* [7], [8].

To better understand the challenges arising in low-inertia power systems, a model for *system-level* analysis is required which correctly captures the physics and bridges the gap between the *system* and *device* level. In this work, we follow a top-down approach based on a first-principles model of a low-inertia power system, including detailed power converter models and their interactions with the power grid [9]. Based on this model we review the main control objectives for power systems and pinpoint discrepancies between the *system-level* specifications and the *device-level* control of power converters. Next, we present a reduced-order model by exploiting the time-scale separation between the DC and AC dynamics via a singular perturbation analysis [10]. The reduced-order model preserves the system structure and approximates the input-output behavior of the power system so that all *system-level* specifications, such as frequency and voltage stability, also apply to the reduced-order model.

Based on the reduced-order model we highlight connections between synchronous machines and power converters on the *device-level* and introduce a virtual oscillator as in [11] and a controller similar to the one used in [12] that matches synchronous generators and power converters on the device level. This controller bridges the gap between *system-level* control objectives and *device-level* control of power converters. Moreover, we present a nonlinear droop control that ensures *frequency* and *device-level* stability.

This paper is structured as follows: In Section II we introduce the model of a low-inertia power system, and in Section III we review the *system-level* control objectives for power systems. In Section IV we present a reduced-order model for low-inertia power systems obtained by using

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singular perturbation theory. This model is used in Section V to bridge the gap between the *system-level* specifications and *device-level* control. In Section VI we present the nonlinear droop control that ensures *frequency* and *device-level* stability for the reduced-order model. Finally, Section VII concludes the paper.

II. Notation and Power System Model

A. Mathematical Notation

We use $\mathbb{R}$ to denote the set of real numbers, $\mathbb{R}_{>0}$ to denote the set of strictly positive real numbers, and $\mathbb{R}_{[a,b]} \coloneqq \{x \in \mathbb{R} \mid a \leq x \leq b\}$. The set $\mathbb{S}^1$ denotes the unit circle, and $\mathbb{T}^n = \mathbb{S}^1 \times \ldots \times \mathbb{S}^1$ denotes the n-torus. For column vectors $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$ we use $(x, y) \coloneqq [x^\top \; y^\top]^\top \in \mathbb{R}^{n+m}$ to denote a stacked vector. Furthermore, I_n denotes the identity matrix of dimension n, $\otimes$ denotes the Kronecker product, and $\|x\|$ denotes the Euclidean norm. Matrices of zeros of dimension $n \times m$ are denoted by $\mathbb{0}_{n\times m}$. Column vectors of zeros and ones of length n are denoted by $\mathbb{0}_n$ and $\mathbb{1}_n$. Given $\theta \in \mathbb{S}^1$ we define the rotation matrix $\mathrm{R}(\theta)$, the 90° rotation matrix j, and the vector $\mathrm{r}(\theta)$ by

$$\mathrm{R}(\theta) \coloneqq \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}, \quad j \coloneqq \mathrm{R}(\pi/2), \quad \mathrm{r}(\theta) \coloneqq \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}.$$

B. Modeling Assumptions and Preliminaries

The model proposed in [9] that is used in this manuscript combines a DC/AC converter model proposed in [12] with a variant of the port-Hamiltonian power system model proposed in [13]. The reader is referred to these references for detailed derivations and discussions of the models. Based on the following assumption all three-phase AC signals can be represented by two dimensional real-valued vectors.

Assumption 1 (Balanced three-phase system) *It is assumed that the electrical components (resistance, inductance, capacitance) of each device have identical values for each phase and that all three-phase signals are balanced.*

Under Assumption 1, a three-phase voltage $v_{abc} \in \mathbb{R}^3$ can be transformed into stationary (α, β) coordinates via the Clarke transformation $T_{abc\to\alpha\beta}$, i.e., $v_{\alpha\beta} = T_{abc\to\alpha\beta} v_{abc} \in \mathbb{R}^2$. In this work, we rewrite the model used in [9] in a reference frame rotating at the constant nominal frequency $\omega_0 \in \mathbb{R}_{>0}$ of the AC signals. In other words, a voltage $v \in \mathbb{R}^2$ in a rotating frame with angle $\theta_r = \omega_0 t \in \mathbb{S}^1$ is defined by $v = \mathrm{R}(\theta_r) v_{\alpha\beta}$. Note that the 90° rotation matrix $j = \mathrm{R}(\pi/2)$ plays the same role that the imaginary unit $\sqrt{-1}$ plays in complex coordinates.

C. Power System Topology

The power system considered in this work consists of n_g synchronous generators, n_c DC/AC converters, n_l constant impedance loads, and n_v AC voltage buses that are interconnected via n_t transmission lines. The topology of the network is described by the (oriented) incidence matrix $E \in \{-1, 1, 0\}^{n_v \times n_t}$, where the voltage buses are the nodes and the transmission lines the edges of the graph induced by E. The k-th voltage bus is connected to transmission lines via ideal transformers with winding ratio $N_k \in \mathbb{R}_{>0}$. The resulting weighted incidence matrix of the AC network is denoted by $\mathcal{E} \in \mathbb{R}^{2n_v \times 2n_t}$ and can be partitioned as follows:

$$\mathcal{E} \coloneqq EN \otimes I_2 = (\mathcal{E}_{v,1}, \ldots, \mathcal{E}_{v,n_v}) = \begin{bmatrix} \mathcal{E}_{t,1} & \ldots & \mathcal{E}_{t,n_t} \end{bmatrix}, \quad (1)$$

where $N = \mathrm{diag}(N_1, \ldots, N_{n_v})$ models the transformer gains. The interconnection of the components is described by indicator matrices $\mathrm{I}_g \in \{1, 0\}^{n_v \times n_g}$, $\mathrm{I}_c \in \{1, 0\}^{n_v \times n_c}$, and $\mathrm{I}_l \in \{1, 0\}^{n_v \times n_l}$. The indicator matrices can be partitioned into column vectors for each device, e.g., $\mathrm{I}_g = \begin{bmatrix} \mathrm{I}_{g,1}, \ldots, \mathrm{I}_{g,n_g} \end{bmatrix}$ and n-th entry of $\mathrm{I}_{g,k}$ is 1 if the k-th synchronous generator is connected to the n-th voltage bus, and 0 otherwise.

D. Dynamical Model of the Transmission System

The transmission network is modeled using the Π-model. In particular, transmission lines are modeled as series RL circuits, and voltage buses are modeled as parallel RC circuits. The input to the network are the currents $i_{\mathrm{in},k} \in \mathbb{R}^2$ injected (or drawn) at each voltage bus $k \in \{1, \ldots, n_v\}$. The state variables of the transmission system model are the line currents $i_t \in \mathbb{R}^{2n_t}$ and bus voltages $v \in \mathbb{R}^{2n_v}$. The current $i_{t,k} \in \mathbb{R}^2$ across the k-th transmission line is given by

$$L_{t,k} \dot{i}_{t,k} = -Z_{t,k} i_{t,k} + \mathcal{E}_{t,k}^\top v, \quad (2)$$

with line inductance $L_{t,k} = I_2 \otimes l_{t,k} \in \mathbb{R}^{2\times2}_{>0}$, line resistance $R_{t,k} = I_2 \otimes r_{t,k} \in \mathbb{R}^{2\times2}_{>0}$, and line impedance $Z_{t,k} = R_{t,k} + j\omega_0 L_{t,k}$. The charge dynamics of a voltage bus $k \in \{1, \ldots, n_v\}$ with voltage $v_k \in \mathbb{R}^2$ are given by

$$C_k \dot{v}_k = -Y_{v,k} v_k + \mathcal{E}_{v,k} i_t + i_{\mathrm{in},k}, \quad (3)$$

where $C_k = I_2 \otimes c_k \in \mathbb{R}^{2\times2}_{>0}$ and $G_k = I_2 \otimes g_k \in \mathbb{R}^{2\times2}_{>0}$ denote the bus capacitance and conductance, and $Y_{v,k} = G_k + j\omega_0 C_k$ denotes the bus admittance. The port variables connecting the power grid to the synchronous generators, DC/AC converters, and loads are the current $i_{\mathrm{in},k}$ flowing in (or out) of each voltage bus $k \in \{1, \ldots, n_v\}$ and the voltage v_k at each voltage bus.

E. Model of a Synchronous Machine

Each synchronous generator is described by the rotor angle $\theta_{g,k} \in \mathbb{S}^1$ relative to the angle of the reference frame θ_r, the absolute angular rotor velocity $\omega_{g,k} \in \mathbb{R}$, and the stator current $i_{g,k} \in \mathbb{R}^2$. For brevity of the presentation, it is assumed that the field current $i_{f,k} \in \mathbb{R}$ is a control input. The k-th synchronous generator is modeled by

$$\dot{\theta}_{g,k} = \omega_{g,k} - \omega_0, \quad (4a)$$

$$M_k \dot{\omega}_{g,k} = -D_k \omega_{g,k} + \tau_{m,k} - \tau_{e,k}, \quad (4b)$$

$$L_{g,k} \dot{i}_{g,k} = -Z_{g,k} i_{g,k} + \mathcal{I}_{g,k}^\top v - v_{\mathrm{ind},k}, \quad (4c)$$

where $\mathcal{I}_{g,k} \coloneqq \mathrm{I}_{g,k} \otimes I_2$, $M_k \in \mathbb{R}_{>0}$ denotes the inertia constant, $D_k \in \mathbb{R}_{>0}$ denotes the damping coefficient, and $\tau_{m,k} \in \mathbb{R}$ is the mechanical torque applied to the rotor. Moreover, $L_{g,k} = I_2 \otimes l_{g,k} \in \mathbb{R}^{2\times2}_{>0}$ and $R_{g,k} = I_2 \otimes r_{g,k} \in \mathbb{R}^{2\times2}_{>0}$ denote the stator inductance and resistance, and $Z_{g,k} =$

$R_{g,k}+j\omega_0 L_{g,k}$ denotes the stator impedance. The electrical torque $\tau_{e,k}:\mathbb{S}^1\times\mathbb{R}^2\times\mathbb{R}\to\mathbb{R}$ is given by

$$\tau_{e,k}(\theta_{g,k},i_{g,k},i_{f,k})=-l_{m,k}i_{f,k}i_{g,k}^\top j\mathrm{r}(\theta_{g,k}), \tag{5}$$

where $l_{m,k}\in\mathbb{R}_{>0}$ is the mutual inductance between the rotor and the stator. Moreover, the voltage $v_{\mathrm{ind},k}:\mathbb{S}^1\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}^2$ induced in the stator is given by

$$v_{\mathrm{ind},k}(\theta_{g,k},\omega_{g,k},i_{f,k})=l_{m,k}i_{f,k}\omega_{g,k}j\mathrm{r}(\theta_{g,k}). \tag{6}$$

F. Model of a 3-Phase DC/AC Converter

In this work, we consider DC/AC converters consisting of a DC link capacitor, a switching block that modulates the DC link capacitor voltage $v_{\mathrm{dc},k}\in\mathbb{R}_{\geq 0}$ into an AC voltage $v_{\mathrm{sw},k}$, and a RL output filter. We assume that a controllable source, e.g., a boost converter connected to photovoltaics and/or a battery is used to supply the DC link capacitor with a DC current $i_{\mathrm{dc},k}\in\mathbb{R}$. By averaging the switched converter dynamics over one switching period an averaged model of the converter is obtained. The switching block is controlled by an averaged modulation signal $m_k\in\mathbb{R}^2_{[-1,1]}$ (see [12]):

$$C_{\mathrm{dc},k}\dot v_{\mathrm{dc},k}=-G_{\mathrm{dc},k}v_{\mathrm{dc},k}-i_{\mathrm{sw},k}, \tag{7a}$$
$$L_{c,k}\dot i_{c,k}=-Z_{c,k}i_{c,k}+\mathcal{I}_{c,k}^\top v-v_{\mathrm{sw},k}, \tag{7b}$$

here $\mathcal{I}_{c,k}:=\mathrm{I}_{c,k}\otimes I_2$, $C_{\mathrm{dc},k}\in\mathbb{R}_{>0}$ is the DC link capacitance, and $G_{\mathrm{dc},k}\in\mathbb{R}_{>0}$ is the DC conductance. Moreover, $L_{c,k}=I_2\otimes l_{s,k}\in\mathbb{R}^{2\times 2}_{>0}$ and $R_{c,k}=I_2\otimes r_{c,k}\in\mathbb{R}^{2\times 2}_{>0}$ denote the output filter inductance and resistance, and $Z_{c,k}=R_{c,k}+j\omega_0 L_{c,k}$ denotes the output filter impedance. The current $i_{\mathrm{sw},k}:\mathbb{R}^2\times\mathbb{R}^2\to\mathbb{R}$ across the switches is given by

$$i_{\mathrm{sw},k}(i_{c,k},m_k)=-\tfrac{1}{2}i_{c,k}^\top m_k. \tag{8}$$

The averaged voltage $v_{\mathrm{sw},k}:\mathbb{R}\times\mathbb{R}^2\to\mathbb{R}^2$ at the AC side of the switching block is given by

$$v_{\mathrm{sw},k}(v_{\mathrm{dc},k},m_k)=\tfrac{1}{2}v_{\mathrm{dc},k}m_k. \tag{9}$$

G. Constant Impedance Loads

Throughout this work, constant impedance loads with load current $i_{l,k}\in\mathbb{R}^2$ are considered. The dynamics are given by

$$L_{l,k}\dot i_{l,k}=-Z_{l,k}i_{l,k}+\mathcal{I}_{l,k}^\top v, \tag{10}$$

where $\mathcal{I}_{l,k}:=\mathrm{I}_{l,k}\otimes I_2$, $L_{l,k}=I_2\otimes l_{l,k}\in\mathbb{R}^{2\times 2}_{>0}$ and $R_{l,k}=I_2\otimes r_{l,k}\in\mathbb{R}^{2\times 2}_{>0}$ denote the load inductance and resistance, and $Z_{l,k}=R_{l,k}+j\omega_0 L_{l,k}$ denotes the load impedance.

H. Dynamic Model of the Power System

By interconnecting the models of the power system components, we obtain a model of the overall power system. We divide the model into its DC and AC variables. The DC state variables are $x=(\theta_g,\omega_g,v_{\mathrm{dc}})\in\mathbb{X}$ with state space $\mathbb{X}:=\mathbb{T}^{n_g}\times\mathbb{R}^{n_g}\times\mathbb{R}^{n_c}$, and the AC state variables are $z=(i_g,i_c,i_l,i_t,v)\in\mathbb{R}^{2n_ac}$ and $n_{\mathrm{ac}}=n_g+n_c+n_l+n_t+n_v$. The control inputs are $u=(\tau_m,i_f,i_{\mathrm{dc}},m)\in\mathbb{U}:=\mathbb{R}^{n_u-2n_c}\times\mathbb{R}^{2n_c}_{[-1,1]}$, and $n_u=2n_g+3n_c$. The full interconnected power system can be expressed as:

$$\dot x=f_{\mathrm{dc}}(x,z,u),\quad \dot z=f_{\mathrm{ac}}(x,z,u). \tag{11}$$

Using the matrices $L=\mathrm{diag}(L_g,L_c,L_l,L_t)$ and $R=\mathrm{diag}(R_g,R_c,R_l,R_t)$ collecting all AC inductances and resistances, as well as the overall interconnection matrix $\mathcal{I}=\begin{bmatrix}\mathcal{I}_g & \mathcal{I}_c & \mathcal{I}_l & \mathcal{E}\end{bmatrix}$, the induced and modulated AC voltages $v_{\mathrm{in}}=(v_{\mathrm{ind}},v_{\mathrm{sw}})$, $n_L=n_{\mathrm{ac}}-n_v$, the matrices $J_n=I_n\otimes j$, $M_z=\mathrm{diag}(L,C)$, $B_z=(-I_{n_g+n_c},\mathbb{0}_{n_l+n_t+n_v\times n_g+n_c})$, and

$$\mathcal{J}_z=\begin{bmatrix}-\omega_0 J_{n_L}L & \mathcal{I}^\top\\ -\mathcal{I} & -\omega_0 J_{n_v}C\end{bmatrix},\quad \mathcal{R}_z=\begin{bmatrix}R & 0\\ 0 & G\end{bmatrix}, \tag{12}$$

the vector field $f_{\mathrm{ac}}:\mathbb{X}\times\mathbb{R}^{2n_{\mathrm{ac}}}\times\mathbb{U}\to\mathbb{R}^{2n_{\mathrm{ac}}}$ is given by

$$f_{\mathrm{ac}}(x,z,u)=M_z^{-1}\big(A_z z+B_z v_{\mathrm{in}}(x,u)\big), \tag{13}$$

where $A_z:=\mathcal{J}_z-\mathcal{R}_z$. Moreover, the vector field $f_{\mathrm{dc}}:\mathbb{X}\times\mathbb{R}^{2n_{\mathrm{ac}}}\times\mathbb{U}\to\mathbb{X}$ can be expressed as follows:

$$\dot\theta_g=\omega_g-\mathbb{1}_{n_g}\omega_0, \tag{14a}$$
$$M\dot\omega_g=-D\omega_g+\tau_m-\tau_e(\theta_g,i_g,i_f), \tag{14b}$$
$$C_{\mathrm{dc}}\dot v_{\mathrm{dc}}=-G_{\mathrm{dc}}v_{\mathrm{dc}}+i_{\mathrm{dc}}-i_{\mathrm{sw}}(i_c,m), \tag{14c}$$

where the nonlinearities are contained in the interconnection terms τ_e and i_{sw}. The induced and modulated AC voltages $v_{\mathrm{in}}:=(v_{\mathrm{ind}},v_{\mathrm{sw}})$ are the outputs of the DC dynamics which act as the input to the AC dynamics, while the AC currents $i_{\mathrm{in}}:=(i_g,i_c,i_l)$ are the outputs of the AC dynamics and inputs to the DC dynamics, i.e., the DC and AC dynamics are interconnected by the port variables v_{in} and i_{in}.

III. Control Specifications

Historically, control specifications for power systems have been defined by focusing on different instability scenarios which arise in practice. This has given rise to a multitude of definitions of power system stability considering different phenomena, time-scales, and system models [1], [14]. Typically, a desired operating point is obtained by solving a dispatch optimization problem which results in setpoints for the voltage magnitude and power injection by each device. Broadly speaking the control objective is to stabilize an equilibrium corresponding to these specifications.

The dynamical model (11) allows to systematically specify a wide range of control specifications. In particular, the specifications with respect to *system-level* stability (see [14]) and *device-level* stability are:

Definition 1 (System-Level Stability)

- ***Frequency Stability:*** *The frequencies of the AC variables z are stable with respect to the desired synchronous frequency ω_0, i.e., the AC variables are stable with respect to the set $\{z\in\mathbb{R}^{n_z}\mid f_{\mathrm{ac}}(x,z,u)=0\}$.*
- ***Angle Stability:*** *The* relative *phase angles of the induced and modulated voltages $v_{\mathrm{in}}=(v_{\mathrm{ind}},v_{\mathrm{sw}})$ converge to a stable equilibrium at which the power injected into the network matches power injection setpoints.*
- ***Voltage Stability:*** *The AC voltage magnitudes $\|v_k\|$ are stable with respect to magnitude setpoints $\hat v_k^\star\in\mathbb{R}_{>0}$.*

Definition 2 (Device-Level Stability)

- ***Rotor Frequency Stability:*** *The rotor speeds ω_g are stable with respect to the synchronous frequency ω_0.*

- ***DC Voltage Stability:*** *The DC voltages $v_{\text{dc},k}$ are stable with respect to setpoints $v^\star_{\text{dc},k} \in \mathbb{R}_{>0}$.*

Observe that for a network of synchronous machines stability of ω_g implies that the relative rotor angles $\delta_{n,k} = \theta_{g,n} - \theta_{g,k}$ converge, i.e., *device-level* stability corresponds to *system-level* stability. In contrast, *device-level* stability of DC/AC converters (7) does not directly correspond to *system-level* stability. This discrepancy will be addressed in Section V-C.

Finally, due to the large-scale nature of the system as well as the destabilizing impact of communication delays and concerns about privacy and cyber-security, only measurements of local states can be used to control each device. Moreover, the angles $\theta_{g,k}$ of the synchronous generators relative to the rotating frame cannot be measured locally.

IV. Model Reduction via Singular Perturbation Analysis

While the model (11) allows to consider a wide range of control objectives, it is also large-scale, highly nonlinear, and contains dynamics on multiple time scales from mechanical and electrical domains. Therefore, in this section, we seek to derive a reduced-order model that is tractable for control design, preserves the system structure, and approximates the full behavior of (11) such that all specifications given in Section III apply to the reduced-order model. To this end, we first show that the AC dynamics are exponentially stable.

Theorem 1 (Exponential Stability of the AC Dynamics) *Given constant DC states $\bar{x}$ and constant inputs $\bar{u}$, the AC dynamics $\dot{z} = f_{\text{ac}}(\bar{x}, z, \bar{u})$ are exponentially stable with respect to $\bar{z}$ such that $f_{\text{ac}}(\bar{x}, \bar{z}, \bar{u}) = 0$.*

Proof: Consider the positive definite matrix M_z, as well as $\bar{x}$, $\bar{z}$, $\bar{u}$ such that $f_{\text{ac}}(\bar{x}, \bar{z}, \bar{u}) = 0$ holds, and the Lyapunov function $V_z = \frac{1}{2}(z - \bar{z})^\top M_z (z - \bar{z})$. Taking the time derivative of V_z along the trajectories of $\dot{z} = f_{\text{ac}}(\bar{x}, z, \bar{u})$ and noting that $f_{\text{ac}}(\bar{x}, \bar{z}, \bar{u}) = 0$ implies $v_{\text{in}}(\bar{x}, \bar{u}) = -A_z \bar{z}$ one obtains $\dot{V}_z = (z - \bar{z})^\top A_z (z - \bar{z})$. Moreover, because $\mathcal{J}_z$ is skew symmetric, i.e., $(z - \bar{z})^\top \mathcal{J}_z (z - \bar{z}) = 0$ for all $(z - \bar{z})$, it follows that $\dot{V}_z = -(z - \bar{z})^\top \mathcal{R}_z (z - \bar{z})$. Because $\mathcal{R}_z$ and M_z are positive definite diagonal matrices, there exists $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}_{>0}$ such that $\alpha_1 \|z - \bar{z}\|^2 \leq V_z \leq \alpha_2 \|z - \bar{z}\|^2$ and $\dot{V}_z \leq -\alpha_1 \|z - \bar{z}\|^2$. By using standard results from Lyapunov theory (e.g. Theorem 4.10 in [15]), it follows that $\dot{z} = f_{\text{ac}}(\bar{x}, z, \bar{u})$ is exponentially stable with respect to $\bar{z}$. ■

Next, we exploit the time-scale separation between the DC and AC variables and define the reduced-order model

$$\dot{\hat{x}} = f_{\text{red}}(\hat{x}, u), \quad \hat{z} = h(\hat{x}, u), \tag{15}$$

where $h(x, u) := -A_z^{-1} B_z v_{\text{in}}(x, u)$ is the steady state of the AC variables and $f_{\text{red}}(x, u) := f_{\text{dc}}(x, h(x, u), u)$. Moreover, we define the time constants $\gamma = (\gamma_{L,1}, \dots, \gamma_{L,n_L}, \gamma_{C,1}, \dots, \gamma_{C,n_C})$ of the n_L inductive and n_C capacitive AC components as:

$$\gamma_{L,k} = \frac{l_k}{\sqrt{r_k^2 + (\omega_0 l_k)^2}}, \quad \gamma_{C,k} = \frac{c_k}{\sqrt{g_k^2 + (\omega_0 c_k)^2}}. \tag{16}$$

The next result exploits the time-scale separation between the DC and AC dynamics to bound the difference between trajectories of the full-order and reduced-order model. To this end, we define $\varepsilon := \max_k \gamma_k \in \mathbb{R}_{>0}$. Because $\gamma_k < \omega_0^{-1}$ holds for all k we obtain the bound $\varepsilon < \omega_0^{-1}$ (see [1] for a similar result).

Theorem 2 (Tikhonov's Theorem) *Consider the dynamic system* (11)*, the reduced-order model* (15)*, and inputs $u = \kappa(\hat{x})$ given by a smooth function $\kappa : \mathbb{X} \to \mathbb{U}$. For any any finite time $t_1 \in \mathbb{R}_{>0}$ there exists a constant $\varepsilon^\star \in \mathbb{R}_{>0}$ such that for all $\epsilon \in \mathbb{R}_{(0,\varepsilon^\star)}$ and all initial conditions $x_0 - \hat{x}_0 = \mathcal{O}(\epsilon)$ and $z_0 - h(\hat{x}_0, \kappa(\hat{x}_0)) = \mathcal{O}(\epsilon)$ the solutions $x(t)$, $z(t)$ of the system* (11) *and the solutions $\hat{x}(t)$ of the reduced-order model* (15) *satisfy*

$$x(t) - \hat{x}(t) = \mathcal{O}(\epsilon), \quad z(t) - h(\hat{x}, \kappa(\hat{x})) = \mathcal{O}(\epsilon) \tag{17}$$

uniformly for $t \in [0, t_1]$.

Proof: In the following we verify the conditions of Theorem 11.1 in [15] which states the desired result. Because the functions f_{ac}, f_{dc}, $h(x, u)$, and f_{red} are smooth the solutions $x(t)$ and $\hat{x}(t)$ are unique, exist, and evolve in a compact subset of $\mathbb{R}^{n_x}$ for all times $t_1 \geq 0$. Next, we rewrite the dynamic model (11) into the standard form of a singular perturbation model. For this, we define $\mathcal{Z} := \operatorname{diag}(R + \omega_0 J_{n_L} L, G + \omega_0 J_{n_v} C)$. By recalling Theorem 1, A_z is exponentially stable, thus it is invertible. Moreover, $A_z - \mathcal{Z}$ is skew symmetric, and it follows that $\mathcal{Z}$ is invertible. Finally, the AC dynamics (13) can be rewritten as

$$\mathcal{Z}^{-1} M_z \dot{z} = \mathcal{Z}^{-1}(A_z z + B_z v_{\text{in}}). \tag{18}$$

The block-diagonal matrix $Z_L^{-1} L \in \mathbb{R}^{2n_L \times 2n_L}$ contains n_L blocks $Z_{L,k}^{-1} L_k \in \mathbb{R}^{2 \times 2}$ which can be decomposed into the time constant $\gamma_{L,k}$ and a unitary matrix U_k as follows

$$Z_{L,k}^{-1} L_k = \frac{l_k}{r_k^2 + (\omega_0 l_k)^2} \begin{bmatrix} r_k & \omega_0 l_k \\ -\omega_0 l_k & r_k \end{bmatrix} = \gamma_{L,k} U_k. \tag{19}$$

The same idea can be used to rewrite the n_v blocks $Y_{v,k}^{-1} C_k$ of the block-diagonal matrix $Y_v^{-1} C \in \mathbb{R}^{2n_v \times 2n_v}$. Using $\mathcal{D}_\gamma := \operatorname{diag}(\gamma_1, \dots, \gamma_{n_{\text{ac}}}) \otimes I_2$ and $U := \operatorname{diag}(U_1, \dots, U_{n_{\text{ac}}})$, the AC dynamics are rewritten into

$$\mathcal{D}_\gamma \dot{z} = U^T \mathcal{Z}^{-1}(A_z z + B_z v_{\text{in}}). \tag{20}$$

By definition of ε there exists multipliers $\lambda_k \in \mathbb{R}_{>0}$ such that γ_k can be rewritten as $\gamma_k = \lambda_k \varepsilon$ for all $k \in \{1, \dots, n_{\text{ac}}\}$ and one obtains the system

$$\varepsilon \dot{z} = (\operatorname{diag}(\lambda_1, \dots, \lambda_{n_{\text{ac}}}) \otimes I_2)^{-1} U^T \mathcal{Z}^{-1}(A_z z + B_z v_{\text{in}}).$$

By letting $\varepsilon \to 0$, which implies $\gamma_k \to 0$ in (20), and performing some manipulations we obtain the algebraic equation $0 = A_z z + v_{\text{in}}(x, \kappa(x))$. Because A_z is invertible the unique solution of this equation is $z = h(x, \kappa(x))$. Next, we define $y_b = z - h(x, \kappa(x))$ and, by treating x as a constant parameter, we obtain the boundary layer system

$$\tfrac{\mathrm{d}}{\mathrm{d}\,\tau} y_b = (\operatorname{diag}(\lambda_1, \dots, \lambda_{n_{\text{ac}}}) \otimes I_2)^{-1} U^T \mathcal{Z}^{-1} A_z y_b. \tag{21}$$

with $\tau = t/\varepsilon$. Because the AC dynamics are linear, exponentially stable by Theorem 1, and U has full rank, they are exponentially stable with respect to the equilibrium $\bar{z} = h(\bar{x}, \kappa(\bar{x}))$ induced by fixed states $\bar{x}$ and the boundary layer system (21) is exponentially stable with respect to the origin. Hence, it follows from Theorem 11.1 in [15] that the trajectories of the power system (11) with $u = \kappa(x)$ are approximated by the trajectories of the reduced system (15) with $u = \kappa(\hat{x})$ and the approximation error on any finite time interval $t \in [0, t_1]$ is of order $\mathcal{O}(\varepsilon)$. ∎

It should be noted that the reduced-order model (15) preserves the structure of the DC dynamics (14) and the AC variables $z = h(x, u)$ are nonlinear outputs of this system. Moreover, if the reduced-order system (15) is exponentially stable the bound (17) holds for $t_1 = \infty$. Finally, in practice, the bound on the approximation error given in Theorem 2 holds for any input signal $u(t)$ which varies on the same time-scale as the DC state variables x.

V. DC/AC Converter Control For System-Level Objectives

As discussed in Section III, *device-level* stability of power converters does not directly correspond to *system-level* stability. In this section, we bridge this gap.

A. Internal Model Control of DC/AC Converters

It was shown in [9], that a necessary condition for synchronous steady-state operation of the power system with frequency ω_0 is that the modulation signal $m_{\alpha\beta,k} \in \mathbb{R}^2$ in the stationary (α, β) frame is given by the internal model $\dot{m}_{\alpha\beta,k} = \omega_0 j m_{\alpha\beta,k}$. In this work, the virtual oscillator $\dot{\theta}_{v,k} = \omega_{c,k}$, with internal state $\theta_{v,k} \in \mathbb{S}^1$, output $m_{\alpha\beta,k} = \mu_k j r(\theta_{v,k})$, and control inputs $\omega_{c,k} \in \mathbb{R}$ and $\mu_k \in \mathbb{R}_{[0,1]}$ for absolute frequency and amplitude is used to satisfy the steady-state conditions of [9]. Relative to the rotating reference frame, the oscillator dynamics and modulation signal m_k of the k-th converter become

$$\dot{\theta}_{c,k} = \omega_{c,k} - \omega_0, \quad m_k = \mu_k j r(\theta_{c,k}). \tag{22}$$

Using this oscillator as internal model results in the state and input vectors $\tilde{x} := (\theta_g, \omega_g, \theta_c, v_{\text{dc}}) \in \tilde{\mathbb{X}}$ and $\tilde{u} = (\tau_m, i_f, i_{\text{dc}}, \omega_c, \mu) \in \tilde{\mathbb{U}}$, where $\tilde{\mathbb{X}} := \mathbb{T}^{n_g} \times \mathbb{R}^{n_g} \times \mathbb{T}^{n_c} \times \mathbb{R}^{n_c}$ and $\tilde{\mathbb{U}} := \mathbb{R}^{n_u - n_c} \times \mathbb{R}^{n_c}_{[0,1]}$. Next, let $\hat{i}_{g,k}(\tilde{x}, \tilde{u})$ and $\hat{i}_{c,k}(\tilde{x}, \tilde{u})$ denote the algebraic model for the currents $i_{g,k}$ and $i_{c,k}$ obtained from the map $h(x, u)$. With a slight abuse of notation, we re-define the electrical torque $\tau_{e,k} : \tilde{\mathbb{X}} \times \tilde{\mathbb{U}} \to \mathbb{R}$ and the switching current $i_{\text{sw},k} : \tilde{\mathbb{X}} \times \tilde{\mathbb{U}} \to \mathbb{R}$ as

$$\tau_{e,k}(\tilde{x}, \tilde{u}) := -l_{m,k}\, i_{f,k}\, \hat{i}_{g,k}(\tilde{x}, \tilde{u})^\top j \mathrm{r}(\theta_{g,k}), \tag{23a}$$

$$i_{\text{sw},k}(\tilde{x}, \tilde{u}) := -\tfrac{1}{2}\, \mu_k\, \hat{i}_{c,k}(\tilde{x}, \tilde{u})^\top j \mathrm{r}(\theta_{c,k}). \tag{23b}$$

Combining the internal model (22) with the reduced-order dynamics (24) results in $\dot{\tilde{x}} = \tilde{f}_{\text{red}}(\tilde{x}, \tilde{u})$ given by

$$\dot{\theta}_g = \omega_g - \mathbb{1}_{n_g} \omega_0, \tag{24a}$$

$$M \dot{\omega}_g = -D \omega_g + \tau_m - \tau_e(\tilde{x}, \tilde{u}), \tag{24b}$$

$$\dot{\theta}_c = \omega_c - \mathbb{1}_{n_g} \omega_0, \tag{24c}$$

$$C_{\text{dc}} \dot{v}_{\text{dc}} = -G_{\text{dc}} v_{\text{dc}} + i_{\text{dc}} - i_{\text{sw}}(\tilde{x}, \tilde{u}). \tag{24d}$$

B. Parallels Between Generators and DC/AC Converters

The energy stored in a synchronous generators is $\frac{1}{2} M_k \omega_k^2$, power is supplied to the synchronous machine by the mechanical torque input τ_m, power is transfered to the grid through the electrical torque τ_e, and power imbalances result in deviations of the frequency ω_g from ω_0. All of today's power system operation has been designed around this inherent property of synchronous generators.

In contrast, the energy stored in the converter is $\frac{1}{2} C_{\text{dc},k} v_{\text{dc},k}^2$, power is supplied by the DC current i_{dc}, power is transfered to the grid by the current i_{sw}, and power imbalances can be observed in the DC voltage $v_{\text{dc},k}$. Moreover, the mechanical torque τ_m, which acts on ω_g, and i_{dc}, which acts on v_{dc}, have the same interpretation. Finally, both i_f and μ act on the voltage magnitudes $\|v_{\text{ind},k}\| = l_{m,k} i_{f,k} \|\omega_{g,k}\|$ and $\|v_{\text{sw},k}\| = \frac{1}{2} \mu_k \|v_{\text{dc},k}\|$. The parallels discussed above are summarized in Table I. These suggest to use a feedback of the form $\omega_c \propto v_{\text{dc}}$ to recover the characteristics of synchronous generators, to use i_{dc} to control v_{dc} (and thereby ω_c), and to use μ to control the magnitude of the AC bus voltages.

C. Control by Model-Matching

For brevity of the presentation we focus on the relationship between power imbalance and frequency. Given a set-point $(\omega_0, v_{\text{dc}}^\star)$ we propose the matching controller

$$\omega_{c,k} = k_{c,k}(v_{\text{dc},k} - v_{\text{dc},k}^\star) + \omega_0 \tag{25}$$

with gain $k_{c,k} \in \mathbb{R}_{>0}$. Because $v_{\text{dc},k}$ reflects the power imbalance, the controller (25) maps power imbalance to frequency deviations just as synchronous generators do. Specifically, using (25) and $K_c = \operatorname{diag}(k_{c,1}, \dots, k_{c,n_c})$ the angle dynamics (24c) become $\dot{\theta}_c = K_c(v_{\text{dc}} - v_{\text{dc}}^\star)$, i.e., the converter model now matches the generator model (24a) and (24b). The matching controller proposed in [12] can be recovered by dropping ω_0 in (25) and using $k_{c,k} = \omega_0 / v_{\text{dc},k}^\star$.

VI. Frequency Stabilization via Decentralized Nonlinear Droop Control

In this section, we focus on frequency stability to illustrate stability analysis and control design based on the reduced order model (24). To this end, consider the torque setpoints $\tau_{m,k}^\star$, the DC current setpoints $i_{\text{dc},k}^\star$, DC voltage setpoints $v_{\text{dc}}^\star$, and the decentralized nonlinear droop control policies

$$\tau_{m,k} = \tau_{m,k}^\star + \omega_{g,k}^{-1} P_{\text{ind},k} - \omega_0^{-1} P_{\text{ind},k}^\star, \tag{26a}$$

$$i_{\text{dc},k} = i_{\text{dc},k}^\star + v_{\text{dc},k}^{-1} P_{\text{sw},k} - (v_{\text{dc},k}^\star)^{-1} P_{\text{sw},k}^\star, \tag{26b}$$

TABLE I

Parallels between sync. generators and DC/AC converters

sync. generator	DC/AC converter	interpretation
$\frac{1}{2} M \omega_g^2$	$\frac{1}{2} C_{\text{dc}} v_{\text{dc}}^2$	energy stored in device
τ_m	i_{dc}	primary power supply
τ_e	i_{sw}	power flow to grid
$\omega_g - \omega_0$	$v_{\text{dc}} - v_{\text{dc},k}^\star$	power imbalance
i_f	μ	AC voltage magnitude

where $P_{\mathrm{sw},k} = -\hat{i}_{c,k}(\tilde{x},\tilde{u})^\top v_{\mathrm{sw},k}$ is the instantaneous active power flowing *out* of the AC side of the converter switching block and *into* the output filter, and $P_{\mathrm{ind},k} = -\hat{i}_{g,k}(\tilde{x},\tilde{u})^\top v_{\mathrm{ind},k}$ is the instantaneous active power flowing *out* of the generator rotor and into the stator. If the losses in the output filter and stator are negligible, $P_{\mathrm{sw},k}$ and $P_{\mathrm{ind},k}$ coincide with the active power injected *into* the power grid. Observe that near steady state (i.e., for $(\omega_g, v_{\mathrm{dc}}) \approx (\omega_0, v_{\mathrm{dc}}^\star)$) the controller (26) resembles an active power droop characteristics (see [16]) that relates the active power injection by the primary control to the frequency ω_g and the DC voltage v_{dc} and, via the matching controller, to the converter frequency ω_c. Finally, we define the vector $\tilde{y} = (\omega_g, v_{\mathrm{dc}}) \in \mathbb{R}^{n_g+n_c}$ and the incremental Lyapunov function

$$V(\tilde{y}) = (\omega_g - \omega_0)^\top M(\omega_g - \omega_0) + (v_{\mathrm{dc}} - v_{\mathrm{dc}}^\star)^\top C_{\mathrm{dc}}(v_{\mathrm{dc}} - v_{\mathrm{dc}}^\star),$$

as well as the level set $\Omega(c) \coloneqq \{\tilde{y} \in \mathbb{R}^{n_g+n_c} | V(\tilde{y}) \leq c\}$. Using the controllers (25) and (26) we obtain the following stability result. We emphasize that only *local* and *measurable* outputs are needed to evaluate the controller for each device.

Theorem 3 (Local Frequency Stabilization)
Consider the power system model (24) *in closed loop with* (25) *and* (26)*. Given $\omega_0 \in \mathbb{R}_{>0}$ and $v_{\mathrm{dc}}^\star \in \mathbb{R}_{>0}^{n_c}$ pick $c \in \mathbb{R}_{>0}$ such that $\Omega(c)$ is a subset of the positive orthant, i.e., $\Omega(c) \subseteq \mathbb{R}_{>0}^{n_g+n_c}$. For any matching gain $k_{c,k} \in \mathbb{R}_{>0}$, any inputs $i_f \in \mathbb{R}$ and $\mu \in \mathbb{R}_{[0,1]}$, and all initial states $\tilde{x}_0$ such that $\tilde{y}_0 \in \Omega(c)$, the frequencies $\omega = (\omega_g, \omega_c)$ are asymptotically stable with respect to ω_0, and the DC voltages $v_{\mathrm{dc}}^\star$ are asymptotically stable with respect to $v_{\mathrm{dc}}^\star$. Moreover, the angles $\theta = (\theta_g, \theta_c)$ converge to constant values.*

Proof: Assuming that $\omega_g \in \mathbb{R}_{>0}^{n_g}$ it follows from (6) that $\omega_{g,k}^{-1} v_{\mathrm{ind},k} = i_{f,k} j \mathrm{r}(\theta_{c,k})$. Substituting into (23a) results in

$$\tau_{e,k} = -\omega_{g,k}^{-1} \hat{i}_{g,k}(\tilde{x},\tilde{u})^\top v_{\mathrm{ind},k} = \omega_{g,k}^{-1} P_{\mathrm{ind},k}, \qquad (27)$$

i.e., by using the algebraic relation (6) the electrical torque can be expressed in terms of the active power injection $P_{\mathrm{ind},k}$ and synchronous generator frequency $\omega_{g,k}$. The same approach can be used to express the current $i_{\mathrm{sw},k}$ in terms of the active power injection and the DC voltage $v_{\mathrm{dc},k}$. Using this reformulation it can be verified that the derivative of the Lyapunov function V along the trajectories of $\dot{\tilde{x}} = \tilde{f}_{\mathrm{red}}(\tilde{x},\tilde{u}), \tilde{u})$ in closed loop with (26), and (25) is given by

$$\dot{V} = -(\omega_g - \omega_0)^\top D(\omega_g - \omega_0) - (v_{\mathrm{dc}} - v_{\mathrm{dc}}^\star)^\top G_{\mathrm{dc}}(v_{\mathrm{dc}} - v_{\mathrm{dc}}^\star).$$

Because D and G_{dc} are positive definite diagonal matrices it follows that the level sets of V are invariant. Using invariance of the level sets of V and $\tilde{y}_0 \in \Omega(c)$ it immediately follows that that $\tilde{y} \in \Omega(c) \subseteq \mathbb{R}_{>0}^{n_g+n_c}$ for all times, i.e., the control laws (26a), and (26b) are well defined for all times. Moreover, using standard results from Lyapunov theory it can be shown that ω_g is asymptotically stable with respect to ω_0, v_{dc} is asymptotically stable with respect to $v_{\mathrm{dc}}^\star$, and (25) implies that ω_c is asymptotically stable with respect to ω_0 for any $k_{c,k} \in \mathbb{R}$. Considering $\omega_g \to \omega_0$ and $v_{\mathrm{dc}} \to v_{\mathrm{dc}}^\star$ the angle dynamics (24a) and (24c) converge to constant angles. ■

Observe that the nonlinear droop control (26) ensure *device-level* and, with the exception of voltage stability, *system-level* stability. Moreover, (26) decouples the frequency dynamics from the inputs i_f and μ. These inputs are a remaining degree of freedom that can be used for voltage magnitude control.

VII. Conclusions

In this paper we studied low-inertia power systems consisting of DC/AC converters and synchronous machines. We present a reduced-order model suitable for control design and analysis that preserves the system structure and bridges the gap between *system-level* control objectives and *device-level* control of power converters by using an internal model and a controller that matches the reduced-order models of DC/AC converters and synchronous machines. Moreover, we provide results on a nonlinear droop control that ensures frequency stability. A detailed investigation of a controller that uses the observations made in Section V-B to fully match the control of DC/AC converters and synchronous generators is the subject of ongoing work.

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