CONTROL SYSTEMS II (REGELSYSTEME II)

PROBLEM SET 2

Objectives:
- Loop-Shaping
- Inverse-Based Controller Design

Background: Sections 2.6 and 2.8 of the Skogestad book

Exercise 1  Inverse-Based Controller Design

With reference to the system in Fig. 1, let

\[ G(s) = G_1(s)G_2(s), \quad G_1(s) = \frac{200}{10s + 1}, \quad G_2(s) = \frac{-s + 1}{s + 1}. \]

a) Let \( G_d(s) = 0 \). Design an “inverse-based controller”

\[ K(s) = \frac{\omega_c}{s} \hat{G}^{-1}(s), \]

where \( \omega_c \) is the crossover frequency and \( \hat{G}(s) \) is the invertible part of the plant, i.e. the minimum phase part with the same magnitude as \( G(s) \). In this example, \( \hat{G}(s) = G_1(s) \). What is an upper bound for the crossover frequency, so that the phase margin is greater than 30°?

For the remainder of the exercise, let

\[ G_d(s) = \frac{20}{100s + 1}. \]

b) What is a lower bound on the crossover frequency \( \omega_c \), so that the controller achieves \( |e(j\omega)| \leq 1 \), for any disturbance \( |d(j\omega)| \leq 1, \forall \omega \)?

c) Propose a crossover frequency such that the controller provides acceptable performance in terms of disturbance rejection and reference tracking.

Hint: Read section 5.6 of the textbook for more details on “limitations imposed by RHP-zeros”.

Figure 1: Block diagram of one degree-of-freedom feedback control system
Exercise 2  Loop-Shaping

With reference to the system in Fig. 1, consider the same system as in Exercise 1, that is, let

\[ G(s) = G_1(s)G_2(s), \quad G_1(s) = \frac{200}{10s + 1}, \quad G_2(s) = \frac{-s + 1}{s + 1}, \quad G_d(s) = \frac{20}{100s + 1}. \]

The objective of this exercise is to validate your results from the previous exercise and try to improve the performance of the controller in the MATLAB environment.

Hint: Use the `bode`, `bodemag`, and `step` functions from the MATLAB Control System Toolbox.

a) The requirement to achieve \(|e(j\omega)| \leq 1\) for a disturbance with \(|d(j\omega)| \leq 1\) is satisfied if

\[ |S(j\omega)| < |G_d^{-1}(j\omega)|, \forall \omega. \]

Plot \(|S(s)|\) and \(|G_d^{-1}(s)|\) for several choices of the crossover frequency \(\omega_c\), between the lower and upper bounds calculated in Exercise 1, to see how this parameter affects the system. Check if the lower and upper bounds on the crossover frequency that you have calculated in the previous exercise hold.

b) The following control objectives are given:

- **Command tracking**: rise time (to reach 90% of the final value) less than 4 seconds and overshoot less than 20%.
- **Disturbance rejection**: the output in response to a unit step disturbance should remain within the range \([-0.5, 0.5]\) at all times, and it should return to 0 as quickly as possible (\(|y(t)|\) should be less than 0.05 after 30 seconds).
- **Input constraints**: \(u(t)\) should remain within \([-1, 1]\) at all times for the above control objectives.

Propose a new controller based on the one degree-of-freedom loop shaping design (see section 2.6.4 of the textbook on “loop shaping for disturbance rejection”) so that the given specifications are fulfilled. Proceed as follows:

- Design a new inverse-based controller, that explicitly takes the dynamical model of the disturbances into account, i.e. \(K_1(s) = G_1(s)^{-1}G_d(s)\). Determine the crossover frequency \(\omega_c\).
- To reduce steady-state errors, add a PI-term to the controller: \(K_2(s) = k_p \left(1 + \frac{1}{s\tau_d}\right) \cdot K_1(s)\). Tune the parameters according to the design objective.
  HINT: Try \(\omega_d = 0.4\omega_c\). This choice limits the phase contribution of the PI-term to approximately \(-22^\circ\). You should achieve an overshoot of \(<50\%\) and an output in response to a unit step disturbance within \([-0.6, 0.6]\).
- To reduce the overshoot, introduce derivative action by adding a lead-compensator to the controller: \(K_3(s) = \frac{T_d}{s^2 + 1} \cdot K_2(s)\), with \(\alpha < 1\).
  HINT: A lead-compensator should ideally have little influence on the crossover frequency, while increasing the corresponding phase and thus the phase margin.

Exercise 3  Weighted Sensitivity

Find the weight \(w_p(s)\), so that the sensitivity function \(S(\omega)\) of the closed-loop system with stable plant is bounded as

\[ |S(\omega)| < \frac{1}{|w_p(s)|}, \]

and the following closed loop objectives are met:

- **Bandwidth** \(\omega_B \geq 0.1\) rad/sec
- **Maximum steady state error** for step setpoint changes is 0.01
- **Phase margin** \(PM \geq 35^\circ\)

Hint: Use the expression for \(w_p\) on slides 3.5-3.7 of the notes in order to determine the parameters \(A\), \(M\) and \(\omega_B^*\).
Exercise 4  Exam 2011, Problem 1a

Consider the one degree-of-freedom feedback control configuration in Fig. 2. The plant is given by

\[ G(s) = \frac{2s - 3}{2s + 4} \]

and the measurement dynamics by

\[ G_m(s) = \frac{2}{2s + 3} \]

![Control Configuration Diagram](image)

Figure 2: One degree-of-freedom feedback control configuration

a) Design an inverse-based controller \( K(s) \) for reference tracking such that

\[ |L(j\omega)| = \frac{\omega_c}{\omega} \quad \forall \omega > 0 \]

holds for the magnitude of the open-loop transfer function \( L(s) \). Keep the crossover frequency \( \omega_c > 0 \) as a positive parameter in the controller design.

b) For what values of \( \omega_c \) will the developed controller achieve closed-loop stability?

c) Consider the reference signal \( r(t) = \frac{1}{2} \sin(\omega t) \). Provide a range of \( \omega_c \) such that acceptable control can be achieved, that is, \( |e(j\omega)| < 1 \) for \( |n(j\omega)| = 0 \) is satisfied.

d) Consider the noise transfer function

\[ G_n(s) = \frac{2}{2s + 3} \]

as depicted in Fig. 2. Show that a choice of \( \omega_c = \frac{1}{4} \) ensures that \( |e(j\omega)| < 1 \) when \( |n(j\omega)| < 1 \) and \( |r(j\omega)| = 0 \).