CONTROL SYSTEMS II (REGELSYSTEME II)

PROBLEM SET 3

Objectives: Bode sensitivity integrals
Background: Sections 5.1.-5.4. of the Skogestad book

Exercise 1 Interpolation Constraints
Consider the stable closed-loop sensitivity function $S(s)$ and complementary sensitivity function $T(s)$ of the plant $G(s)$.

a) Prove the following interpolation constraints for a possible RHP-zero $z$ or RHP-pole $p$ of $G(s)$

\[
G(z) = 0 \Rightarrow T(z) = 0, \quad S(z) = 1
\]

\[
G^{-1}(p) = 0 \Rightarrow T(p) = 1, \quad S(p) = 0
\]

b) What can you say in each of the above cases for possible RHP-zeros or RHP-poles in the loop transfer function $L(s)$.

Exercise 2 Stable Plant with no RHP Zeros
Consider the open-loop transfer function

\[
L_1(s) = \frac{1}{(s + 1)(s + 2)}
\]

The plant is stable and minimal phase. What is the value of the Bode sensitivity integral for this plant? Verify your result by plotting the magnitude of the closed-loop sensitivity function and by numerical integration.

Hint: You may use the Matlab command trapz for trapezoidal numerical integration.

Exercise 3 Stable Plant with RHP Zero
Consider the open-loop transfer function

\[
L_2(s) = \frac{4 - s}{(s + 1)(s + 2)}
\]

The plant is stable but has a RHP zero. What is the value of the Bode sensitivity integral for this plant? Verify your result by plotting the weighted magnitude of the sensitivity function and by numerical integration.
**Exercise 4  Unstable Plant with no RHP Zeros**

Consider the open-loop transfer function

\[ L_3(s) = \frac{5}{(s - 1)(s + 2)}. \]

The plant has no RHP zeros but is unstable. What is the value of the Bode sensitivity integral for this plant? Verify your result by plotting the weighted magnitude of the sensitivity function and by numerical integration.

**Exercise 5  Unstable Plant with RHP Zero**

Consider the open-loop transfer function

\[ L_4(s) = \frac{0.5(4 - s)}{s - 1}. \]

The plant is unstable and has a RHP zero. What is the value of the Bode sensitivity integral for this plant? Verify your result by plotting the weighted magnitude of the sensitivity function and by numerical integration.